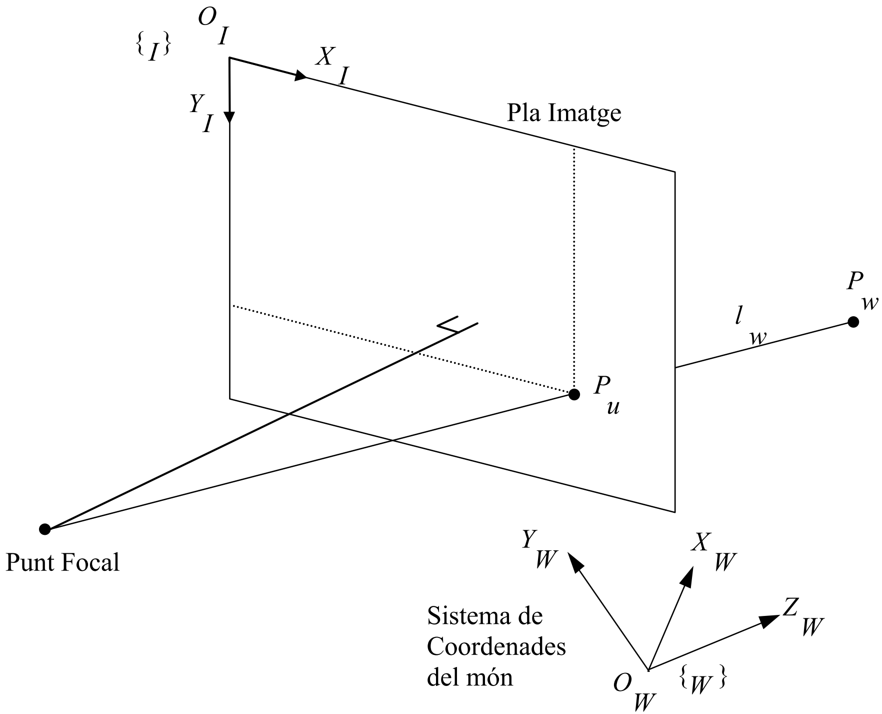
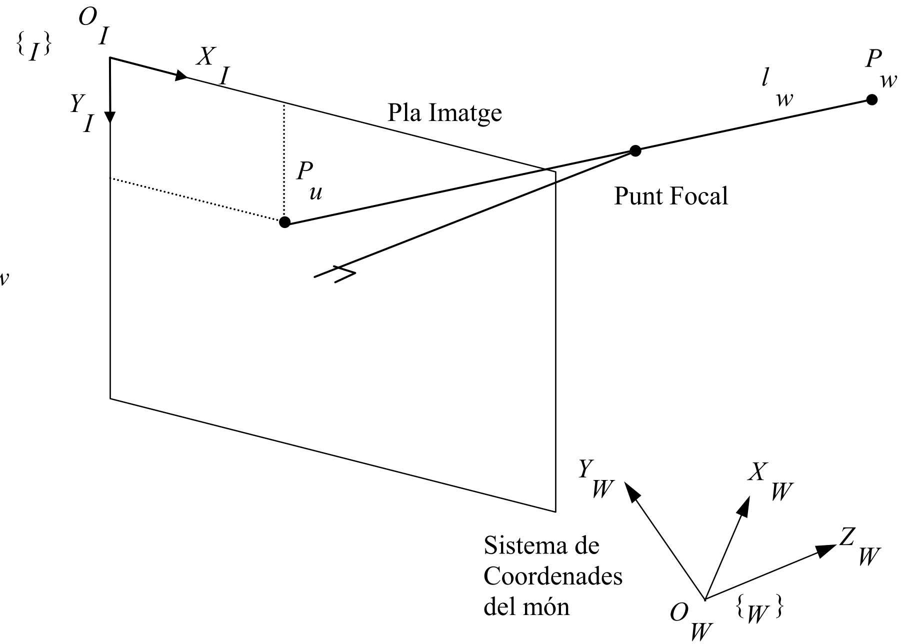
TEMA 5

Calibració de Càmeres

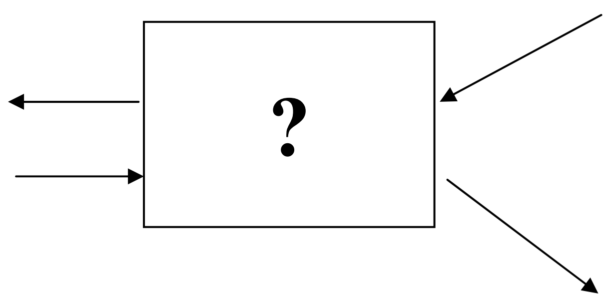
Calibració de càmeres





$$^{I}P_{u} = \begin{pmatrix} ^{I}X_{u} \\ ^{I}Y_{u} \\ 1 \end{pmatrix}$$

En píxels

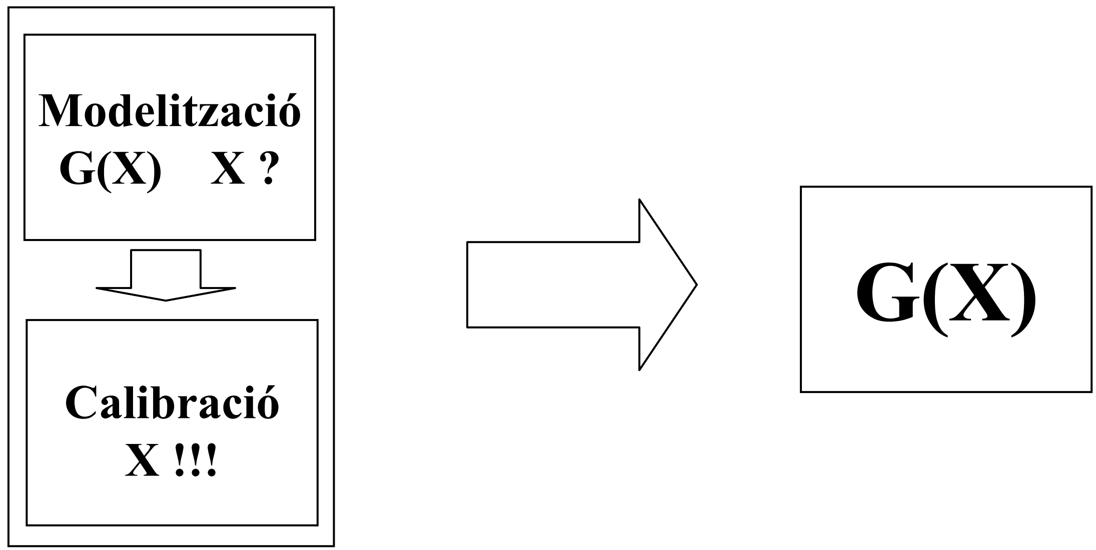


$$^{W}P_{w} = \begin{pmatrix} ^{W}X_{w} \\ ^{W}Y_{w} \\ ^{W}Z_{w} \\ 1 \end{pmatrix}$$

$$^{W}l_{w} = \begin{pmatrix} ^{W}X_{w} \\ ^{W}Y_{w} \\ ^{W}Z_{w} \\ 0 \end{pmatrix}$$

En mm.

Calibració de càmeres



Modelització:

- Especificar l'equació matemàtica que regirà el comportament de la càmara.
- Aquesta equació dependrà d'un conjunt de paràmetres.
- Aproximació matemàtica al model físic real de la càmara.

Calibració:

- Obtenció del valor dels paràmetres del model.

Mètode de Hall - Model de la càmera

Suposem que la llum es projecta al pla imatge de forma lineal.

$$\begin{pmatrix} s\,{}^{I}X_u \\ s\,{}^{I}Y_u \\ s \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & A_{34} \end{pmatrix} \begin{pmatrix} {}^{W}X_w \\ {}^{W}Y_w \\ {}^{W}Z_w \\ 1 \end{pmatrix}$$

Matriu definida a partir d'un factor d'escala

Solució única fixant una component qualsevol a la unitat.

$$\begin{pmatrix} s\,{}^{I}X_u \\ s\,{}^{I}Y_u \\ s \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & 1 \end{pmatrix} \begin{pmatrix} {}^{W}X_w \\ {}^{W}Y_w \\ {}^{W}Z_w \\ 1 \end{pmatrix}$$

Mètode de Hall - Calibració

$$\begin{pmatrix} s\,{}^{I}X_u \\ s\,{}^{I}Y_u \\ s \end{pmatrix} = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} \\ A_{21} & A_{22} & A_{23} & A_{24} \\ A_{31} & A_{32} & A_{33} & 1 \end{pmatrix} \begin{pmatrix} {}^{W}X_w \\ {}^{W}Y_w \\ {}^{W}Z_w \\ 1 \end{pmatrix}$$

$${}^{I}X_u = \frac{A_{11}\,{}^{W}X_w + A_{12}\,{}^{W}Y_w + A_{13}\,{}^{W}Z_w + A_{14}}{A_{31}\,{}^{W}X_w + A_{32}\,{}^{W}Y_w + A_{33}\,{}^{W}Z_w + 1}$$

$${}^{I}Y_u = \frac{A_{21}\,{}^{W}X_w + A_{22}\,{}^{W}Y_w + A_{23}\,{}^{W}Z_w + A_{24}}{A_{31}\,{}^{W}X_w + A_{32}\,{}^{W}Y_w + A_{33}\,{}^{W}Z_w + 1}$$

$$A_{11}\,{}^{W}X_w - A_{31}\,{}^{I}X_u\,{}^{W}X_w + A_{12}\,{}^{W}Y_w - A_{32}\,{}^{I}X_u\,{}^{W}Y_w + A_{13}\,{}^{W}Z_w - A_{33}\,{}^{I}X_u\,{}^{W}Z_w + A_{14} = {}^{I}X_u$$

$$A_{21}\,{}^{W}X_w - A_{31}\,{}^{I}Y_u\,{}^{W}X_w + A_{22}\,{}^{W}Y_w - A_{32}\,{}^{I}Y_u\,{}^{W}Y_w + A_{23}\,{}^{W}Z_w - A_{33}\,{}^{I}Y_u\,{}^{W}Z_w + A_{24} = {}^{I}Y_u$$

Mètode de Hall - Calibració

$$A_{11}\,{}^{W}X_w - A_{31}\,{}^{I}X_u\,{}^{W}X_w + A_{12}\,{}^{W}Y_w - A_{32}\,{}^{I}X_u\,{}^{W}Y_w + A_{13}\,{}^{W}Z_w - A_{33}\,{}^{I}X_u\,{}^{W}Z_w + A_{14} = {}^{I}X_u$$

$$A_{21}\,{}^{W}X_w - A_{31}\,{}^{I}Y_u\,{}^{W}X_w + A_{22}\,{}^{W}Y_w - A_{32}\,{}^{I}Y_u\,{}^{W}Y_w + A_{23}\,{}^{W}Z_w - A_{33}\,{}^{I}Y_u\,{}^{W}Z_w + A_{24} = {}^{I}Y_u$$

$$A = \begin{pmatrix} A_{11} & A_{12} & A_{13} & A_{14} & A_{21} & A_{22} & A_{23} & A_{24} & A_{31} & A_{32} & A_{33} \end{pmatrix}^{\mathrm{T}}$$

Tenim 11 incògnites i cada punt 2D ens dona 2 equacions.

Necessitem com a mínim 6 punts. Més punts implica més precissió

$$QA = B$$

$$Q_{2i-1} = \begin{pmatrix} {}^{W}X_{wi} & {}^{W}Y_{wi} & {}^{W}Z_{wi} & 1 & 0 & 0 & 0 & 0 & -{}^{I}X_{ui}\,{}^{W}X_{wi} & -{}^{I}X_{ui}\,{}^{W}Y_{wi} & -{}^{I}X_{ui}\,{}^{W}Z_{wi} \end{pmatrix}$$

$$Q_{2i} = \begin{pmatrix} 0 & 0 & 0 & 0 & {}^{W}X_{wi} & {}^{W}Y_{wi} & {}^{W}Z_{wi} & 1 & -{}^{I}Y_{ui}\,{}^{W}X_{wi} & -{}^{I}Y_{ui}\,{}^{W}Y_{wi} & -{}^{I}Y_{ui}\,{}^{W}Z_{wi} \end{pmatrix}$$

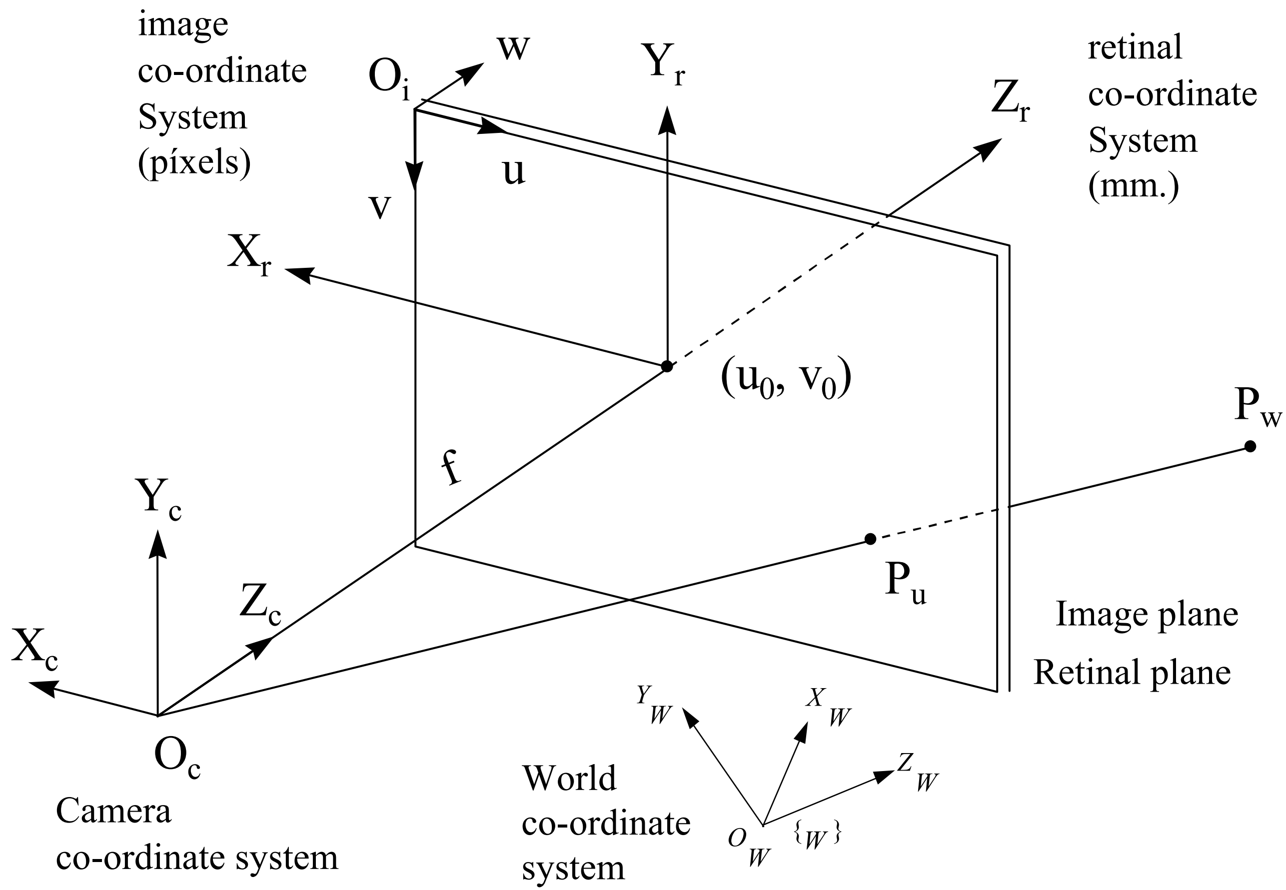
$$B_{2i-1} = \begin{pmatrix} {}^{I}X_{ui} \end{pmatrix}$$

$$B_{2i} = \begin{pmatrix} {}^{I}Y_{ui} \end{pmatrix}$$

Aleshores, es pot obtenir una solució per la matriu A, utilitzant la pseudoinversa.

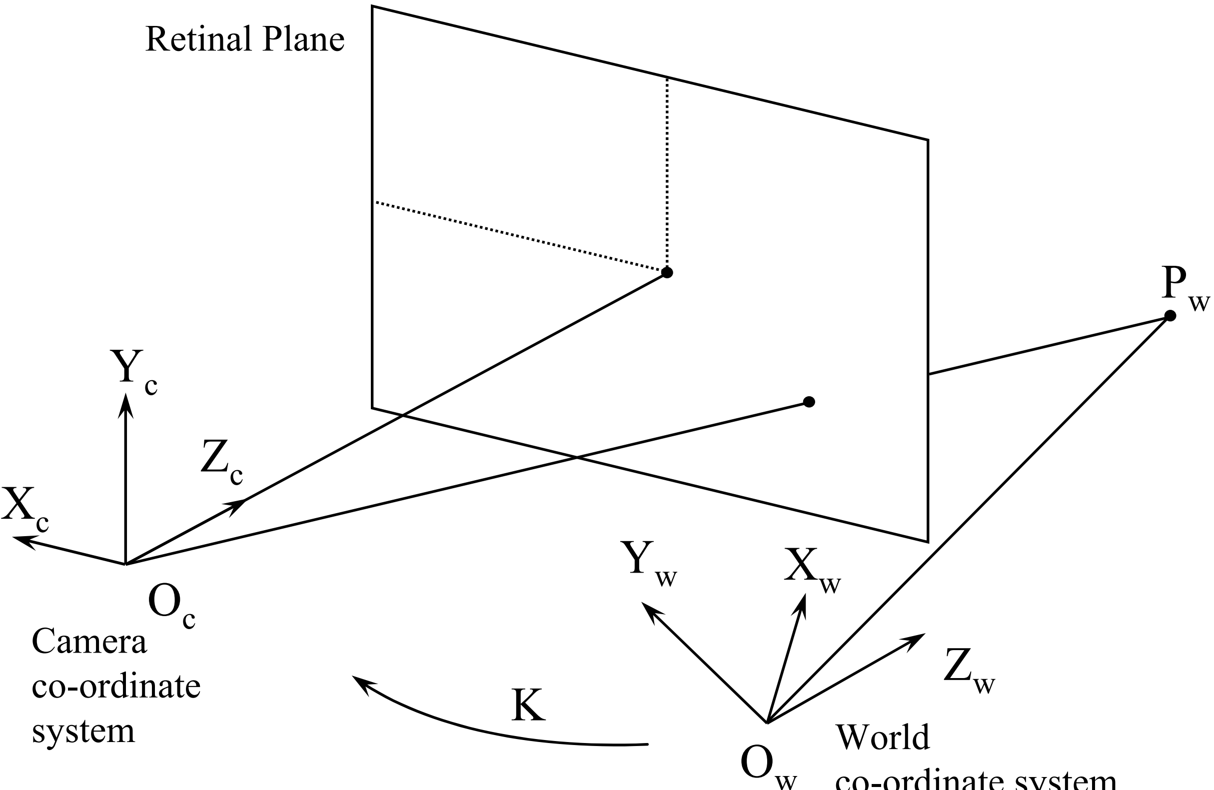
$$A = Q^{-1}B \quad \Rightarrow \quad \boxed{A = \left(Q^{t}Q\right)^{-1} Q^{t}B}$$

Mètode de Faugeras - El model pinhole



- Extrinsic parameters: Model the situation and orientation of the camera with respect to a world co-ordinate system.
- Intrinsic parameters: Model the behaviour of the internal geometry and the optical characteristics of the camera.

Mètode de Faugeras - Paràmetres extrínsecs



$$^{C}R_{W} = Rot(X,\alpha)\cdot Rot(Y,\beta)\cdot Rot(Z,\gamma)$$

$$^{C}R_{W} = \begin{pmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{pmatrix}$$

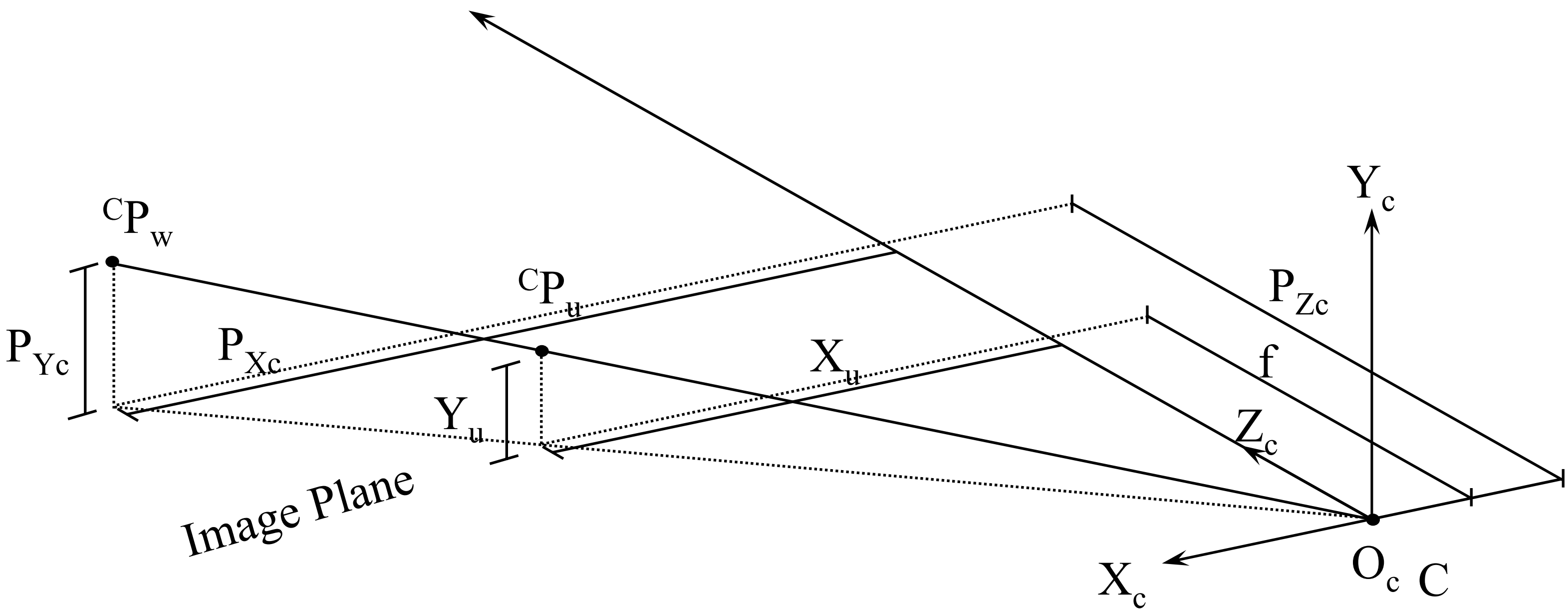
$$^{C}T_{W} = \begin{pmatrix} t_X \\ t_Y \\ t_Z \end{pmatrix}$$

$$\begin{pmatrix} ^{C}X_w \\ ^{C}Y_w \\ ^{C}Z_w \end{pmatrix} = {}^{C}R_{W} \begin{pmatrix} ^{W}X_w \\ ^{W}Y_w \\ ^{W}Z_w \end{pmatrix} + {}^{C}T_{W}$$

$$\begin{pmatrix} ^{C}P_w \\ 1 \end{pmatrix} = {}^{C}K_{W} \begin{pmatrix} ^{W}P_w \\ 1 \end{pmatrix}$$

$$^{C}K_{W} = \begin{pmatrix} ^{C}R_{W\,3x3} & ^{C}T_{W\,3x1} \\ 0_{1x3} & 1 \end{pmatrix}$$

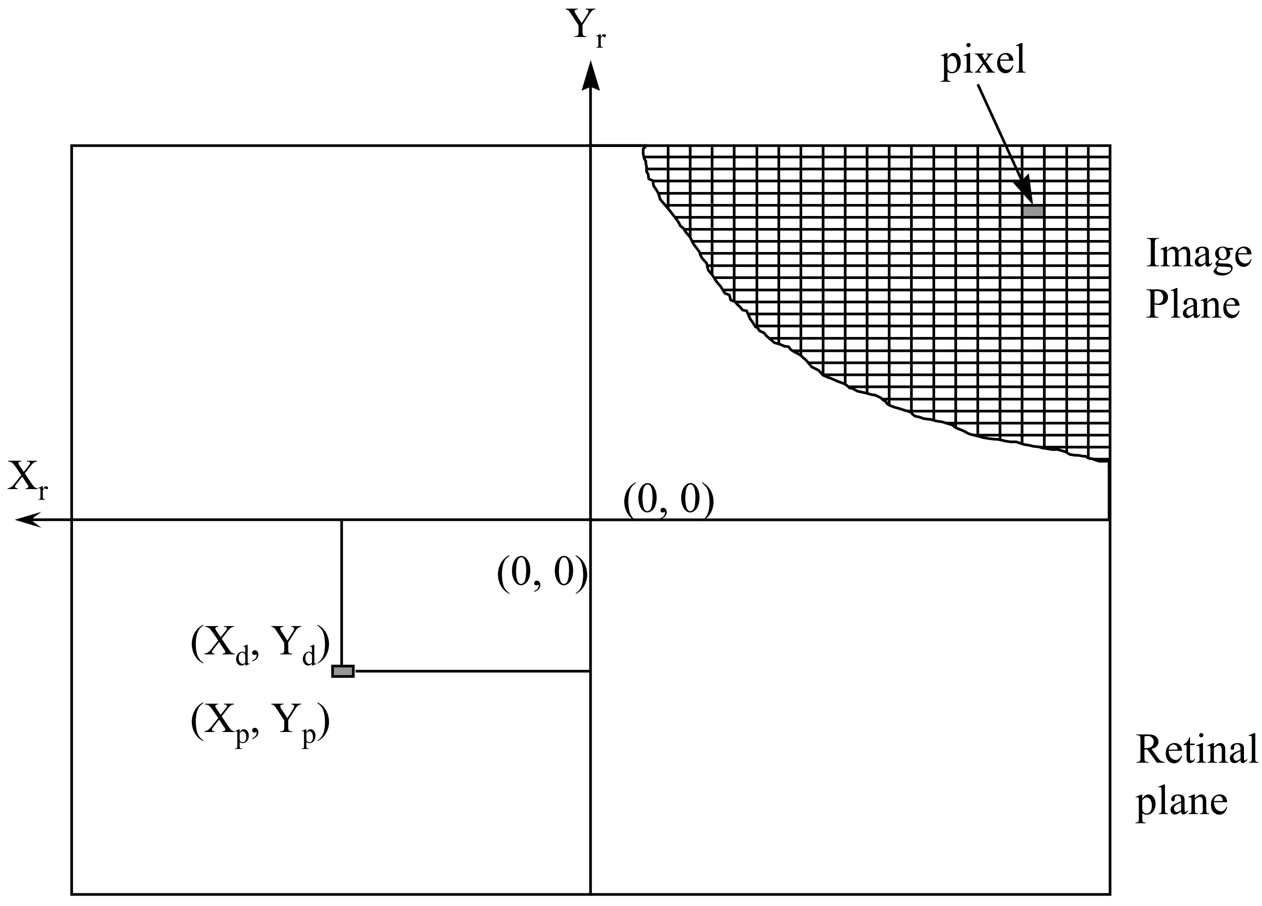
Mètode de Faugeras - Paràmetres intrínsecs - Projecció ideal



$$^{C}X_{u} = f\,\frac{^{C}X_{w}}{^{C}Z_{w}}$$

$$^{C}Y_{u} = f\,\frac{^{C}Y_{w}}{^{C}Z_{w}}$$

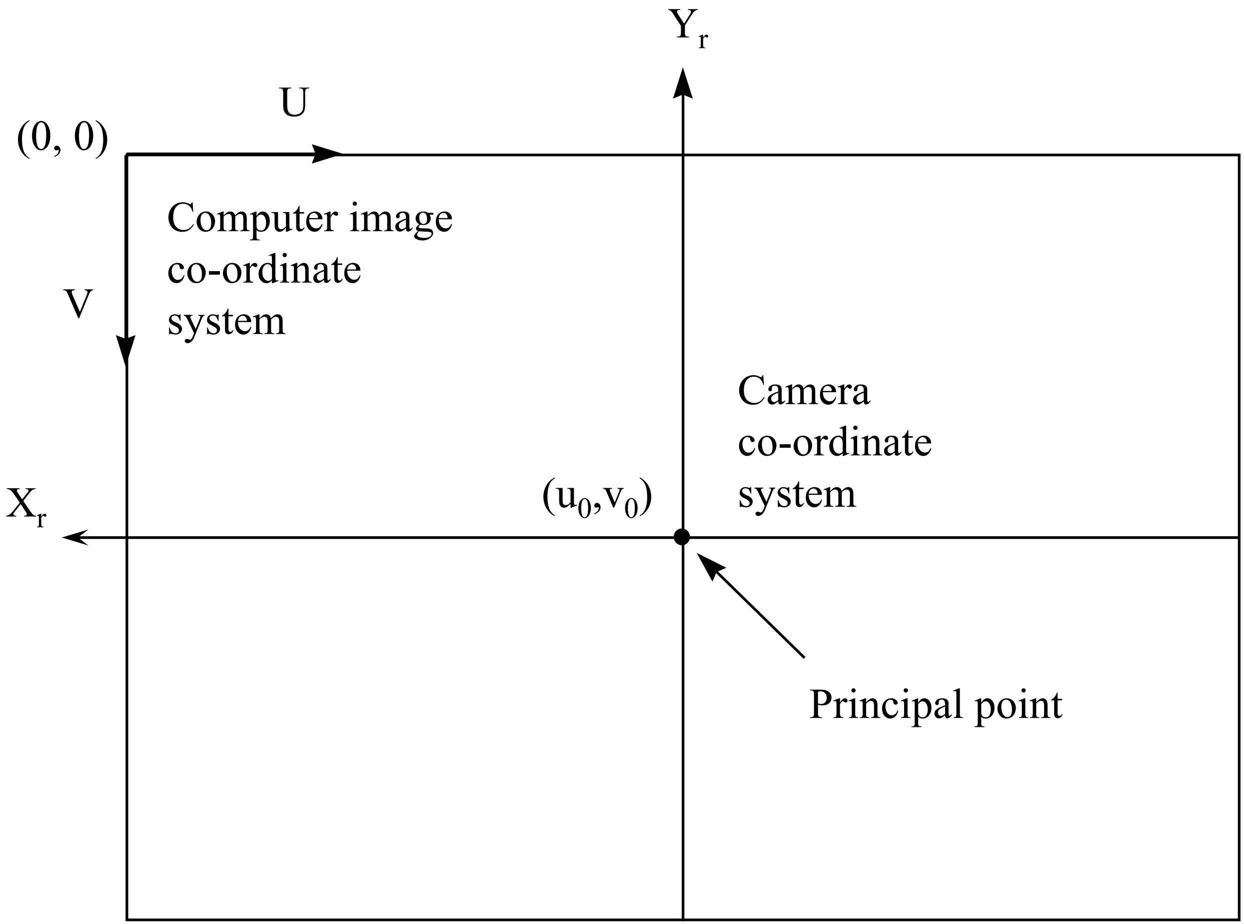
Mètode de Faugeras - Paràmetres intrínsecs - Conversió píxel



$$^{R}X_{d} = k_{u}\ ^{C}X_{u}$$

$$^{R}Y_{d} = k_{v}\ ^{C}X_{u}$$

Mètode de Faugeras - Paràmetres intrínsecs - Punt Principal



$$^{I}X_d = -\,^{R}X_d + u_0$$

$$^{I}Y_d = -\,^{R}X_d + v_0$$

Mètode de Faugeras - Model de la càmera

($\mathbf{X_w}$, $\mathbf{Y_w}$, $\mathbf{Z_w}$) 3D object point with respect to world co-ordinate system

Affine transformation.
Modelled parameters: **R, T**

($\mathbf{X_c}$, $\mathbf{Y_c}$, $\mathbf{Z_c}$) 3D object point with respect to camera co-ordinate system

Perspective transformation.
Modelled parameter: **f**

($\mathbf{X_u}$, $\mathbf{Y_u}$) Ideal projection on the retinal plane

Pixel adjustment
Modelled parameters: $\mathbf{k_u}$, $\mathbf{k_v}$

($\mathbf{X_p}$, $\mathbf{Y_p}$) Real projection on the image plane

Adaptation to the computer image buffer
Modelled parameters: $\mathbf{u_0}$, $\mathbf{v_0}$

($\mathbf{X_i}$, $\mathbf{Y_i}$) Real projection on the image plane

Mètode de Faugeras - Paràmetres intrínsecs

$$ {}^{C}X_u = f\,\frac{{}^{C}X_w}{{}^{C}Z_w} \qquad {}^{R}X_d = k_u\,{}^{C}X_u \qquad {}^{I}X_d = -{}^{R}X_d + u_0 $$

$$ {}^{C}Y_u = f\,\frac{{}^{C}Y_w}{{}^{C}Z_w} \qquad {}^{R}Y_d = k_v\,{}^{C}X_u \qquad {}^{I}Y_d = -{}^{R}X_d + v_0 $$

$$ {}^{I}X_u = -k_u f\,\frac{{}^{C}X_w}{{}^{C}Z_w} + u_0 $$

$$ {}^{I}Y_u = -k_v f\,\frac{{}^{C}Y_w}{{}^{C}Z_w} + v_0 $$

$$ \begin{pmatrix} s\,{}^{I}X_u \\ s\,{}^{I}Y_u \\ s \end{pmatrix} = \begin{pmatrix} \alpha_u & 0 & u_0 & 0 \\ 0 & \alpha_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} {}^{C}X_w \\ {}^{C}Y_w \\ {}^{C}Z_w \\ 1 \end{pmatrix} $$

$$ \alpha_u = -fk_u $$

$$ \alpha_v = -fk_v $$

Mètode de Faugeras - Model de la càmera

Intrínsecs Extrínsecs

$$\begin{pmatrix} s\,{}^{I}X_u \\ s\,{}^{I}Y_u \\ s \end{pmatrix} = \begin{pmatrix} \alpha_u & 0 & u_0 & 0 \\ 0 & \alpha_v & v_0 & 0 \\ 0 & 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} r_{11} & r_{12} & r_{13} & t_x \\ r_{21} & r_{22} & r_{23} & t_y \\ r_{31} & r_{32} & r_{33} & t_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} {}^{W}X_w \\ {}^{W}Y_w \\ {}^{W}Z_w \\ 1 \end{pmatrix}$$

$$A = \begin{pmatrix} \alpha_u r_1 + u_0 r_3 & \alpha_u t_x + u_0 t_z \\ \alpha_v r_2 + v_0 r_3 & \alpha_v t_y + v_0 t_z \\ r_3 & t_z \end{pmatrix}$$

Mètode de Faugeras - Calibració

$$A_1\,{}^W P_w + A_{14} - {}^I X_u \left(A_3\,{}^W P_w + A_{34} \right) = 0$$

$$A_2\,{}^W P_w + A_{24} - {}^I Y_u \left(A_3\,{}^W P_w + A_{34} \right) = 0$$

$${}^I X_u = \frac{A_1}{A_{34}}\,{}^W P_w + \frac{A_{14}}{A_{34}} - \frac{A_3}{A_{34}}\,{}^W P_w\,{}^I X_u$$

$${}^I Y_u = \frac{A_2}{A_{34}}\,{}^W P_w + \frac{A_{24}}{A_{34}} - \frac{A_3}{A_{34}}\,{}^W P_w\,{}^I Y_u$$

$${}^I X_u = T_1\,{}^W P_w + C_1 - T_2\,{}^W P_w\,{}^I X_u$$

$${}^I Y_u = T_3\,{}^W P_w + C_2 - T_2\,{}^W P_w\,{}^I Y_u$$

$$T_1 = \frac{r_3}{t_z} u_0 + \frac{r_1}{t_z} \alpha_u \qquad C_1 = u_0 + \frac{t_x}{t_z} \alpha_u$$

$$T_2 = \frac{r_3}{t_z}$$

$$T_3 = \frac{r_3}{t_z} v_0 + \frac{r_2}{t_z} \alpha_v \qquad C_2 = v_0 + \frac{t_y}{t_z} \alpha_v$$

Mètode de Faugeras - Calibració

$$B = QX$$

$$X = \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ C_1 \\ C_2 \end{pmatrix} \qquad Q = \begin{pmatrix} & & \cdots & & \\ {}^W P_{wi}^{\,t} & -{}^I X_{ui}\,{}^W P_{wi}^{\,t} & 0_{1x3} & 1 & 0 \\ 0_{1x3} & -{}^I Y_{ui}\,{}^W P_{wi}^{\,t} & {}^W P_{wi}^{\,t} & 0 & 1 \\ & & \cdots & & \end{pmatrix} \qquad B = \begin{pmatrix} \cdots \\ {}^I X_{ui} \\ {}^I Y_{ui} \\ \cdots \end{pmatrix}$$

$$\boxed{X = \left(Q^t Q\right)^{-1} Q^t B}$$

Mètode de Faugeras - Calibració - Obtenció dels paràmetres

$$T_1 = \frac{r_3}{t_z} u_0 + \frac{r_1}{t_z} \alpha_u \qquad C_1 = u_0 + \frac{t_x}{t_z} \alpha_u$$

$$T_2 = \frac{r_3}{t_z}$$

$$T_3 = \frac{r_3}{t_z} v_0 + \frac{r_2}{t_z} \alpha_v \qquad C_2 = v_0 + \frac{t_y}{t_z} \alpha_v$$

$$\left\| r_3 \right\| = 1 \quad \Rightarrow \quad t_z = \frac{1}{\left\| T_2 \right\|}$$

Mètode de Faugeras - Calibració - Obtenció dels paràmetres

$$T_1 = \frac{r_3}{t_z} u_0 + \frac{r_1}{t_z} \alpha_u \qquad C_1 = u_0 + \frac{t_x}{t_z} \alpha_u$$

$$T_2 = \frac{r_3}{t_z}$$

$$T_3 = \frac{r_3}{t_z} v_0 + \frac{r_2}{t_z} \alpha_v \qquad C_2 = v_0 + \frac{t_y}{t_z} \alpha_v$$

$$v_1 v_2 = \|v_1\| \|v_2\| \cos \alpha$$

$$v_1 \wedge v_2 = \|v_1\| \|v_2\| \sin \alpha$$

$$r_i r_j^t = 0 \quad i \neq j$$

$$r_i r_j^t = 1 \quad i = j$$

$$r_i \wedge r_j = 1 \quad i \neq j$$

$$r_i \wedge r_j = 0 \quad i = j$$

$$u_0 = \frac{T_1 T_2^t}{\|T_2\|^2} \qquad v_0 = \frac{T_1 T_3^t}{\|T_2\|^2}$$

$$\alpha_u = \frac{\|T_1^t \wedge T_2^t\|}{\|T_2\|^2} \qquad \alpha_v = \frac{\|T_2^t \wedge T_3^t\|}{\|T_2\|^2}$$

Mètode de Faugeras - Calibració - Obtenció dels paràmetres

$$T_1 = \frac{r_3}{t_z} u_0 + \frac{r_1}{t_z} \alpha_u \qquad C_1 = u_0 + \frac{t_x}{t_z} \alpha_u$$

$$T_2 = \frac{r_3}{t_z}$$

$$T_3 = \frac{r_3}{t_z} v_0 + \frac{r_2}{t_z} \alpha_v \qquad C_2 = v_0 + \frac{t_y}{t_z} \alpha_v$$

$$v_1 v_2 = \|v_1\| \|v_2\| \cos \alpha$$

$$v_1 \wedge v_2 = \|v_1\| \|v_2\| \sin \alpha$$

$$r_i r_j^t = 0 \quad i \neq j$$

$$r_i r_j^t = 1 \quad i = j$$

$$r_i \wedge r_j = 1 \quad i \neq j$$

$$r_i \wedge r_j = 0 \quad i = j$$

$$r_1 = \frac{\|T_2\|}{\|T_1^t \wedge T_2^t\|} \left(T_1 - \frac{T_1 T_2^t}{\|T_2\|^2} T_2 \right)$$

$$r_2 = \frac{\|T_2\|}{\|T_2^t \wedge T_3^t\|} \left(T_3 - \frac{T_2 T_3^t}{\|T_2\|^2} T_2 \right)$$

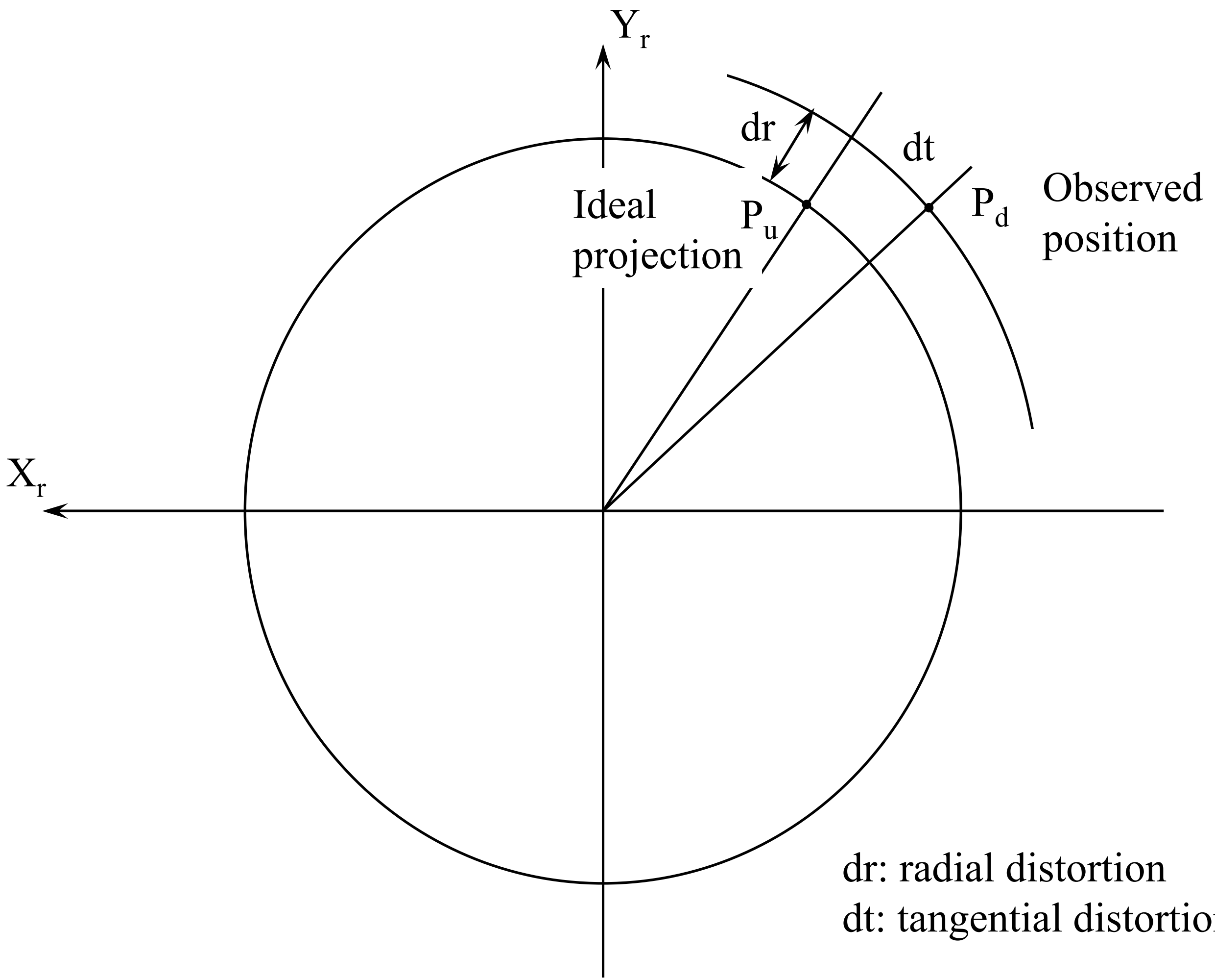
$$r_3 = \frac{T_2}{\|T_2\|}$$

$$t_x = \frac{\|T_2\|}{\|T_1^t \wedge T_2^t\|} \left(C_1 - \frac{T_1 T_2^t}{\|T_2\|^2} \right)$$

$$t_y = \frac{\|T_2\|}{\|T_2^t \wedge T_3^t\|} \left(C_2 - \frac{T_2 T_3^t}{\|T_2\|^2} \right)$$

$$t_z = \frac{1}{\|T_2\|}$$

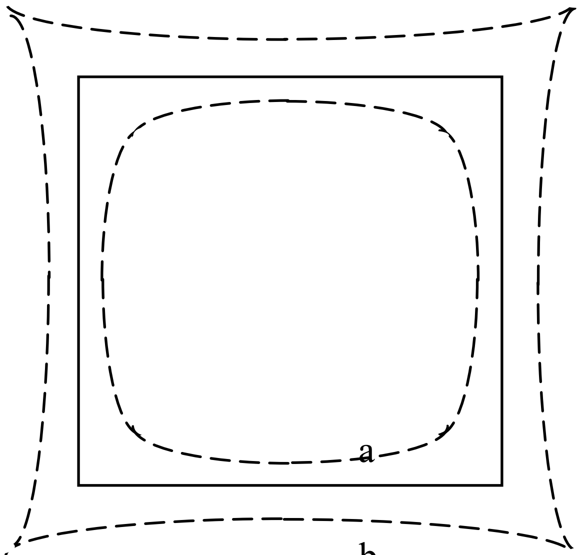
Mètode de Faugeras amb distorsió de les lens



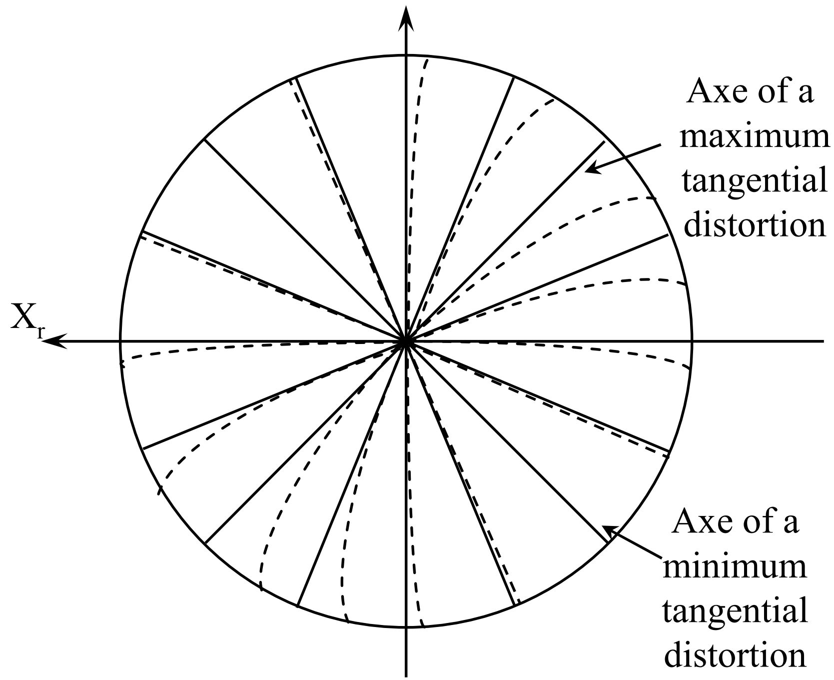
dr: radial distortion
dt: tangential distortion

Mètode de Faugeras amb distorsió de les lens

Efecte de la distorsió radial



Efecte de la distorsió tangencial



La distorsió radial es la que afecta més, i normalment és la única que es té en compte en la majoria de models.

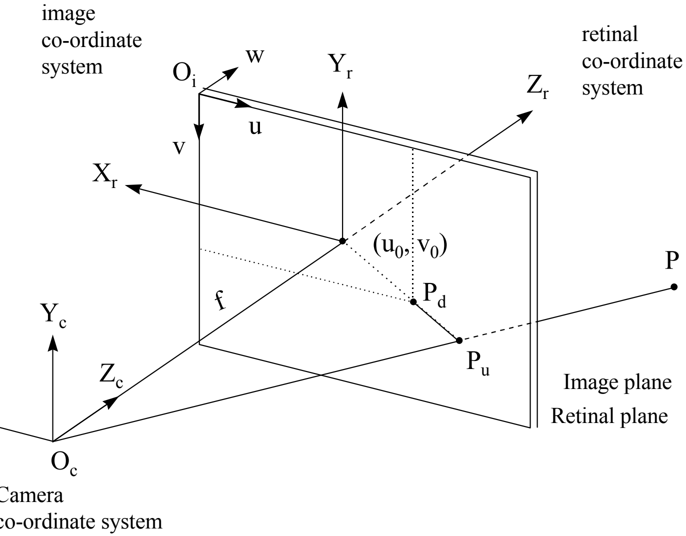
Mètode de Faugeras - Model amb distorsió radial - Paràmetres intrínsecs

$$D_x = {}^{C}X_d\left(k_1 r^2 + k_2 r^4 + \cdots\right)$$

$$D_y = {}^{C}Y_d\left(k_1 r^2 + k_2 r^4 + \cdots\right)$$

$$r = \sqrt{{}^{C}X_d^{\ 2} + {}^{C}Y_d^{\ 2}}$$

$\Rightarrow$ k_1 es el factor més important



$$\frac{X_u}{f} = \frac{P_{Xc}}{P_{Zc}} \qquad \frac{Y_u}{f} = \frac{P_{Yc}}{P_{Zc}}$$

$$X_u = X_d + D_x \qquad Y_u = Y_d + D_y$$

$$D_x = X_d k_1 r^2 \qquad D_y = Y_d k_1 r^2 \qquad r = \sqrt{X_d^{\ 2} + Y_d^{\ 2}}$$

$$X_p = k_u X_d \qquad Y_p = k_v Y_d$$

$$X_i = -X_p + u_0 \qquad Y_i = -Y_p + v_0$$

Mètode de Faugeras - Model amb distorsió radial

$(\mathbf{X_w}, \mathbf{Y_w}, \mathbf{Z_w})$ 3D object point with respect to world co-ordinate system

Affine transformation. Modelled parameters: **R, T**

$(\mathbf{X_c}, \mathbf{Y_c}, \mathbf{Z_c})$ 3D object point with respect to camera co-ordinate system

Perspective transformation. Modelled parameter: **f**

$(\mathbf{X_u}, \mathbf{Y_u})$ Ideal projection on the retinal plane

Radial lens distortion. Modelled parameter: $\mathbf{k_1}$

$(\mathbf{X_d}, \mathbf{Y_d})$ Real projection on the retinal plane

Pixel adjustment Modelled parameters: $\mathbf{k_u}, \mathbf{k_v}$

$(\mathbf{X_p}, \mathbf{Y_p})$ Real projection on the image plane

Adaptation to the computer image buffer Modelled parameters: $\mathbf{u_0}, \mathbf{v_0}$

$(\mathbf{X_i}, \mathbf{Y_i})$ Real projection on the image plane

Mètode de Faugeras - El model amb distorsió radial

$$f\frac{{}^{C}X_{w}}{{}^{C}Z_{w}} = {}^{C}X_{d} + k_{1}r^{2}\,{}^{C}X_{d}$$

$$f\frac{{}^{C}Y_{w}}{{}^{C}Z_{w}} = {}^{C}Y_{d} + k_{1}r^{2}\,{}^{C}Y_{d}$$

$${}^{C}X_{d} = \frac{\left({}^{I}X_{d} - u_{0}\right)}{-k_{u}}$$

$${}^{C}Y_{d} = \frac{\left({}^{I}Y_{d} - v_{0}\right)}{-k_{v}}$$

$$r = \sqrt{X_{d}^{\,2} + Y_{d}^{\,2}}$$

$$\begin{pmatrix} {}^{C}X_{w} \\ {}^{C}Y_{w} \\ {}^{C}Z_{w} \\ 1 \end{pmatrix} = {}^{C}K_{W}\begin{pmatrix} {}^{W}X_{w} \\ {}^{W}Y_{w} \\ {}^{W}Z_{w} \\ 1 \end{pmatrix}$$

El model és NO LINEAL ⇨ Calibració per mètodes iteratius