The SLAM problem: a survey

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Abstract. This paper surveys the most recent published techniques in the field of Simultaneous Localization and Mapping (SLAM). In particular it is focused on the existing techniques available to speed up the process, with the purpose to handel large scale scenarios. The main research field we plan to investigate is the filtering algorithms as a way of reducing the amount of data. It seems that almost all the current approaches can not perform consistent maps for large areas, mainly due to the increase of the computational cost and due to the uncertainties that become prohibitive when the scenario becomes larger.

Keywords. SLAM, Kalman filter, Particle Filter, Expectation Maximization

Introduction

Simultaneous Localization and Mapping (SLAM) also known as Concurrent Mapping and Localization (CML) is one of the fundamental challenges of robotics, dealing with the necessity of building a map of the environment while simultaneously determining the location of the robot within this map. The aim of this paper is to survey the advantages and disadvantages of current available techniques, to compare and contrast them, and finally to identify gaps, i.e. possible new research directions or further improvements. This survey is conducted as the starting point of a bigger project, which involves computer vision in SLAM (Visual SLAM) in underwater scenarios.

SLAM is a process by which a mobile robot can build a map of an environment and at the same time use this map to deduce its location. Initially, both the map and the vehicle position are not known, the vehicle has a known kinematic model and it is moving through the unknown environment, which is populated with artificial or natural landmarks. A simultaneous estimate of both robot and landmark locations is required. The SLAM problem involves finding appropriate representation for both the observation and the motion models [1]. In order to do so, the vehicle must be equipped with a sensorial system capable of taking measurements of the relative location between landmarks and the vehicle itself.

Several research groups and researchers have worked and are currently working in SLAM, and the most commonly used sensors can be categorized into laser-based, sonar-based, and vision-based systems. Additional sensorial sources are used to better perceive robot state information and the outside world [2], such as, compasses, infrared technology and Global Positioning System (GPS). However, all these sensors carry certain errors, often referred to as measurement noise, and also have several range limitations making necessary to navigate through the environment, for instance, light and sound cannot penetrate walls.

Laser ranging systems are accurate active sensors. Its most common form operates on the time of flight principle by sending a laser pulse in a narrow beam towards the object and measuring the time taken by the pulse to be reflected off the target and returned to the sender. *Sonar-based systems* are fast and provide measurements and recognition capacities with amounts of information similar to vision, but with the lack of appearance data. However, its dependence on inertial sensors, such as odometers, implies that a small error can have large effects on later position estimates [2]. On the other hand, *Vision systems* are passive, they have long range and high resolution, but the computation cost is considerably high and good visual features are more difficult to extract and match. Vision is used to estimate the 3D structure, feature location and robot pose, for instance by means of stereo camera pairs or monocular cameras with structure from motion recovery.

Further classification can be made in terms of working environment, for instance, ground indoor, ground outdoor, air-borne or underwater. Most of the work done so far focuses on ground and mainly indoor environments [3] [4] [5] [6], only few papers deal with airborne applications [7] [8] and a few more present the SLAM in underwater conditions and they generally work with acoustic data [9]. Recently, there is a growing interest in SLAM for underwater scenarios [10], in which vision plays an important role [11] [12] [13], in most cases combined with other sensory systems to acquire both depth and appearance information of the scene, for instance, acoustic or inertial sensors.

The representation of the observation and the motion models is generally performed by computing its prior and posterior distributions using probabilistic algorithms, which are briefly described in section 1. These algorithms are strongly influenced by the data measurement and association, which are presented in section 2. Existing literature is classified in section 3, listing the main advantages and disadvantages of each group. Finally, section 4 summarizes the main ideas and an overall discussion is given.

1. Solutions to the SLAM Problem : Filters in SLAM

Robotic map-building can be traced back to 25 years ago, and since the 1990s probabilistic approaches (i.e. Kalman Filters (KF), Particle Filters (PF) and Expectation Maximization (EM)) have become dominant in SLAM. The three techniques are mathematical derivations of the recursive Bayes rule. The main reason for this probabilistic techniques popularity is the fact that robot mapping is characterized by uncertainty and sensor noise, and probabilistic algorithms tackle the problem by explicitly modeling different sources of noise and their effects on the measurements [2].

1.1. Kalman Filters and its variations (KF)

Kalman filters are Bayes filters that represent posteriors using Gaussians, i.e. unimodal multivariate distributions that can be represented compactly by a small number of parameters. KF SLAM relies on the assumption that the state transition and the measurement functions are linear with added Gaussian noise, and the initial posteriors are also Gaussian. There are two main variations of KF in the state-of-the-art SLAM: the Extended Kalman Filter (EKF) and its related Information Filtering (IF) or Extended IF (EIF). The

EKF accommodates the nonlinearities from the real world, by approximating the robot motion model using linear functions. Several existing SLAM approaches use the EKF [3] [5] [14] [15]. The IF is implemented by propagating the inverse of the state error covariance matrix. There are several advantages of the IF filter over the KF. Firstly, the data is filtered by simply summing the information matrices and vector, providing more accurate estimates [16]. Secondly, IF are more stable than KF [17]. Finally, EKF is relatively slow when estimating high dimensional maps, because every single vehicle measurement generally effects all parameters of the Gaussian, therefore the updates requires prohibitive times when dealing with environments with many landmarks [18].

However, IF have some important limitations, a primary disadvantage is the need to recover a state estimate in the update step, when applied to nonlinear systems. This step requires the inversion of the information matrix. Further matrix inversions are required for the prediction step of the information filters. For high dimensional state spaces the need to compute all these inversions is generally believed to make the IF computationally poorer than the Kalman filter. In fact, this is one of the reasons why the EKF has been vastly more popular than the EIF [19]. These limitations do not necessarily apply to problems in which the information matrix possesses structure. In many robotics problems, the interaction of state variables is local; as a result, the information matrix may be sparse. Such sparseness does not translate to sparseness of the covariance. Information filters can be thought of as graphs, where states are connected whenever the corresponding off-diagonal element in the information matrix is non-zero. Sparse information matrices correspond to sparse graphs. Some algorithms exist to perform the basic update and estimation equations efficiently for such fields [20] [21], in which the information matrix is (approximately) sparse, and allow to develop an extended information filter that is significantly more efficient than both Kalman filters and non sparse Information Filter.

The Unscented Kalman Filter (UKF) [22] addresses the approximation issues of the EKF and the linearity assumptions of the KF. KF performs properly in the linear cases, and is accepted as an efficient method for analytically propagating a Gaussian Random Variable (GRV) through a linear system dynamics. For non linear models, the EKF approximates the optimal terms by linearizing the dynamic equations. The EKF can be viewed as a first-order approximation to the optimal solution. In these approximations the state distribution is approximated by a GRV, which then is propagated analytically through the first-order linearization of the nonlinear system. These approximations can introduce large errors in the true posterior mean and covariance, which may lead sometimes to divergence of the filter. In the UKF the state distribution is again represented by a GRV, but is now specified using a minimal set of carefully chosen sample points. These sample points completely capture the true mean and covariance of the GRV, and when propagated through the true non-linear system, captures the posterior mean and covariance accurately to the 3rd order for any nonlinearity. In order to do that, the unscented transform is used.

One of the main drawbacks of the EKF and the KF implementation is the fact that for long duration missions, the number of landmarks will increase and, eventually, computer resources will not be sufficient to update the map in real-time. This scaling problem arises because each landmark is correlated to all other landmarks. The correlation appears since the observation of a new landmark is obtained with a sensor mounted on the mobile robot and thus the landmark location error will be correlated with the error in the vehicle location and the errors in other landmarks of the map. This correlation is of fundamental importance for the long-term convergence of the algorithm, and needs to be maintained for the full duration of the mission. The Compressed Extended Kalman Filter (CEKF) [23] algorithm significantly reduces the computational requirement without introducing any penalties in the accuracy of the results. A CEKF stores and maintains all the information gathered in a local area with a cost proportional to the square of the number of landmarks in the area. This information is then transferred to the rest of the global map with a cost that is similar to full SLAM but in only one iteration.

The advantage of KF and its variants is that provides optimal Minimum mean-square Error (MMSE) estimates of the state (robot and landmark positions), and its covariance matrix seems to converge strongly. However, the Gaussian noise assumption restricts the adaptability of the KF for data association and number of landmarks.

1.2. Particle Filter based methods (PF)

PF, also called the sequential Monte-Carlo (SMC) method, is a recursive Bayesian filter that is implemented in Monte Carlo simulations. It executes SMC estimation by a set of random point clusters ('particles') representing the Bayesian posterior. In contrast to parametric filters (e.g., KF), PF represents the distribution by a set of samples drawn from this distribution, what makes it capable of handling highly nonlinear sensors and non-Gaussian noise. However, this ability produces a growth in computational complexity on the state dimension as new landmarks are detected, becoming not suitable for real time applications [24]. For this reason, PF has only been successfully applied to localization, i.e. determining position and orientation of the robot, but not to map-building, i.e. landmark position and orientation; therefore, there are no important papers using PF for the whole SLAM framework, but there exist few works that deal with the SLAM problem using a combination of PF with other techniques, for instance, the FastSLAM [24] and the fastSLAM2.0 [25]. FastSLAM takes advantage of an important characteristic of the SLAM problem (with known data association): landmark estimates are conditionally independent given the robot's path [26]. FastSLAM algorithm decomposes the SLAM problem into a robot localization problem, and a collection of landmark estimation problems that are conditioned on the robot pose estimate. A key characteristic of FastSLAM is that each particle makes its own local data association. In contrast, EKF techniques must commit to a single data association hypothesis for the entire filter. In addition FastSLAM uses a particle filter to sample over robot paths, which requires less memory usage and computational time than a standard EKF or KF.

1.3. Expectation Maximization based methods (EM)

EM estimation is a statistical algorithm that was developed in the context of maximum likelihood (ML) estimation and it offers an optimal solution, being an ideal option for map-building, but not for localization. The EM algorithm is able to build a map when the robot's pose is known, for instance, by means of expectation [27]. EM iterates two steps: an expectation step (E-step), where the posterior over robot poses is calculated for a given map, and maximization step (M-step), in which the most likely map is calculated given these pose expectations. The final result is a series of increasingly accurate maps. The main advantage of EM with respect to KF is that it can tackle the correspondence problem (data association problem) surprisingly well [2]. This is possible thanks to the fact that it localizes repeatedly the robot relative to the present map in the E-step, generating various hypotheses as to where the robot might have been (different possible correspondences). In the latter M-step, these correspondences are translated into features in the map, which then get reinforced in the next E-step or gradually disappear. However, the need to process the same data several times to obtain the most likely map makes it inefficient, not incremental and not suitable for real-time applications [28]. Even using discrete approximations, when estimating the robot's pose, the cost grows exponentially with the size of the map, and the error is not bounded; hence the resulting map becomes unstable after long cycles. These problems could be avoided if the data association was known [29], what is the same, if the E-step was simplified or eliminated. For this reason, EM usually is combined with PF, which represents the posteriors by a set of particles (samples) that represent a guess of the pose where the robot might be. For instance, some practical applications use EM to construct the map (only the M-step), while the localization is done by different means, i.e. using PF-based localizer to estimate poses from odometer readings [2].

2. Measuring and Data Association

The most fundamental key topic into all SLAM solutions is the *data association* problem, which arises when landmarks cannot be uniquely identified, and due to this the number of possible hypotheses may grow exponentially, making absolutely unviable the SLAM for large areas. Data association in SLAM can be simply presented as a feature correspondence problem, which identifies two features observed in different positions and different points in time as being from the same physical object in the world. Two common applications of such data association are to match two successive scenes and to close a loop of a long trajectory when a robot comes to the starting point of the trajectory again.

So far most computer vision approaches only uses 2D information to perform data association, but in underwater scenarios this data association is more complicated due to more significant levels of distortion and noise. Therefore, in order to succeed when solving the correspondence problem very robust features are necessary, even under weak lighting conditions or under different points of view. The use of vision sensors offers the possibility to extract landmarks considering 2D and 3D information [11] [30], hence more robust features can be selected.

Feature recognition, tracking and 3D reconstruction are important steps that feed the measurements to the SLAM framework. Feature tracking is the problem of estimating the locations of features in an image sequence (for instance, Harris corner detector and Random Sample Consensus (RANSAC), Scale Invariant Feature Transform (SIFT) [15] or Speeded-up Robust Features (SURF) [31]). 3D reconstruction is the problem of obtaining the 3D coordinates and the camera pose using two or more 2D images (for instance, by using epipolar geometry and fundamental matrix). Fortunately, recent advances in computer vision techniques and, more precisely, in feature extraction enable the usage of high-level vision-based landmarks (complex and natural structures) in contrast to early attempts using low-level features (e.g., vertical edges, line segments, etc.) and artificial beacons. However, the use of vision has several limitations and practical difficulties, for example, several assumptions about the environment must be done in order to simplify the problem (similarly to what humans do using prior knowledge to make decisions on what their eyes see); in regions without texture or with repeating structures there is no method of finding true matches; and even if this matching problem is solved, images are always noisy.

The *loop-closing* problem (a robot turns back to the starting point of its trajectory) requires successful identification of revisited landmarks to build a consistent map in large scale environments. Due to accumulated errors along the trajectory (drift), the reconstructed map is not consistent, i.e., the loop of the trajectory is not closed properly. Correct data association is required to uniquely identify the landmarks corresponding to previously seen ones, from which loop-closing can be detected. Then, different techniques are applied to correct the map, for example, Kalman smoother-based (used by most of the current solutions to the SLAM problem) and EM-based techniques [27].

3. Classification: Pros and Cons

From the previous section, it seems clear that few works has been published on underwater SLAM [5] [13] and even less on underwater visual SLAM [30]. Most of the underwater approaches use sonar or other non visual sensory systems. There exist various V-SLAM approaches for terrestrial applications [3] [4] [14] [15], most of them deal with the uncertainties by using Kalman Filters (KF) and its variation Extended Kalman Filters (EKF) [3] [14] [15], and another group of papers uses some improved information filter [16] [17] [20] [21], i.e. sparse expanded information filter (SEIF).

It seems obvious that almost non of the current approaches can perform consistent maps for large areas, mainly due to the increase on computational cost and on the uncertainties. Therefore this is possibly the most important issue that needs to be improved. Some recent publications tackle the problem by using multiple maps, or sub-maps that are lately used to build a larger global map [4] [32] [33]. However these methods rely considerably on assuming proper data association, which is another important issue that needs to be improved. Table 1 provides a list of advantages and disadvantages of different SLAM strategies in terms of the method used to deal with uncertainties.

Essentially, the most challenging methods not still solved are the ones enabling large-scale implementations in increasingly unstructured environments, i.e. underwater, and especially in situations where other current solutions are unavailable or unreliable. According to the bibliographical survey, SLAM solutions could be improved either by formulating more efficient and consistent to large scenarios filtering algorithms, and solving in a very robust way the data association problem. For the first case, different filters applied into the SLAM framework must be studied, for instance the compressed extended Kalman Filter (CEKF), the Unscented Kalman Filter (UKF) or the information filters(IF/EIF). The second issue is currently solved using SIFT and SURF, which seem to be considerably good solutions for the data association problem, however they become computationally expensive when dealing with high dimensional maps.

4. Discussion

This survey allows to find the most interesting filtering techniques and identify many of its particularities. These filtering strategies are Kalman Filter (KF), Information Filter

Table 1. List of advantages and disadvantages of filtering approaches applied into the SLAM framework.

Pros	*Cons*
Kalman Filter and Extended KF (KF/EKF) [3] [5] [14] [15]	
- high convergence	- Gaussian assumption
- handle uncertainty	- slow in high dimensional maps
Compressed Extended KF (CEKF) [23]	
- reduced uncertainty	- require very robust features
- reduction of memory usage	- data association problem
- handle large areas	- require multiple map merging
- increase map consistency	
Information Filters (IF) [16] [17]	
- stable and simple	- data association problem
- accurate	- may need to recover a state estimates
- fast for high dimensional maps	- in high-D is computationally expensive
Particle Filter (PF) [24] [25]	
- handle nonlinearities	- growth in complexity
- handle non-Gaussian noise	
Expectation Maximization (EM) [16] [27]	
- optimal to map building	- inefficient, cost growth
- solve data association	- unstable for large scenarios
	- only successful in map building

(IF), Unscented Kalman Filter (UKF) and Compressed Kalman Filter (CKF). A general classification of the current filtering strategies is given, contrasting the pros and cons.

The most interesting outcome from the survey is that for large scenarios, or maps with high population of landmarks, the CKF seems to be better as compared to other methods. When dealing with these kind of maps, the state vector and its associated covariance matrix keeps growing with the quantity of landmarks observed. This growth makes the mathematical operations more complex and increases dramatically the time consumption, i.e. the computational cost. The strategy used by the CKF to compute local KFs and then update its output to a global map seems really consistent, because it only needs to handle with small amounts of data during the local iteration process.

As always, there is room for additional improvement. Further areas of research that have been identified encouraging future investigation are:

1. *Test and adapt all methods to non-linear problems.* this means to implement and test linear models, and then improve them by implementing its extended formulations, for instance, Extended Kalman Filter (EKF), Extended Information Filter(EIF) and Compressed Extended Kalman Filter (CEKF).
2. *Find new solutions for non-gaussian noise problems.* Although gaussian noise is assumed in all models presented so far, not always reflects the problems of the real world. It seems that UKF could handle with different types of noise, but this topic has not been investigated in deep yet.
3. *Develop Visual SLAM algorithms useful for navigation purposes on underwater vehicles and for 3D reconstruction of the seafloor.* In fact, this is the ultimate destination of this work, which should contribute to improve current underwater

vehicles and underwater research. This survey is a first step to this whole work, bringing insightful information on filtering techniques. As far as data association problem is concerned, it is well known that it is completely dependent on the ability to find proper features, which in underwater scenarios is a considerably critical task, mainly due to the enormous amount of noise, the dynamism of the environment and the lack of significant landmarks. Obviously if this landmark detection is not robust enough there won't be possibility for the SLAM to work. For this reason, the use of vision to define landmarks seems to open a wide range of different possibilities. In addition a vision system is considerably less expensive than other sensorial systems, for instance, sonar. The main drawback lay in its high computational cost to extract and match features. Further problems that needs to be addressed from the limitations that arises from using computer vision in SLAM are the dynamism of the robot environment, i.e. changes of location of other agents in the environment, creates a big challenge, because it introduces inconsistent sensor measurements and because there are almost no algorithms that can dealt with mapping in dynamic environments.

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