Visual servoing

Practical session

François Chaumette
IRISA / INRIA Rennes

1 Introduction

When we are interested in controlling the 6 dof of an eye-in-hand system to realize a positioning task, we recall that a control law minimizing the error between the current visual features $\mathbf{s}(t)$ and the desired one $\mathbf{s}^*$ is given by:

$$\mathcal{T}_c = -\lambda \, \widehat{\mathbf{L}_\mathbf{s}^T}^+ (\mathbf{s}(t) - \mathbf{s}^*) \tag{1}$$

where

- $\mathcal{T}_c = (V_x, V_y, V_z, \omega_x, \omega_y, \omega_z)^T$ is the camera kinematics screw sent to the low-level robot controller;
- λ is a proportional gain tuning the convergence speed:
- $\widehat{\mathbf{L}_\mathbf{s}^T}$ is an approximation of the interaction matrix related to $\mathbf{s}$ (defined such that $\dot{\mathbf{s}} = \mathbf{L}_\mathbf{s}^T \mathcal{T}$). $\mathbf{L}_\mathbf{s}^T$ is of dimension $k \times 6$ where k is the number of visual features selected in $\mathbf{s}$ and has to be of full rank 6 to control the 6 dof of the system. Usaually, $\widehat{\mathbf{L}_\mathbf{s}^T}$ is chosen as:

$$\widehat{\mathbf{L}_\mathbf{s}^T} = \mathbf{L}_\mathbf{s}^T{}_{|\mathbf{s}=\mathbf{s}^*} \quad \text{or} \quad \widehat{\mathbf{L}_\mathbf{s}^T} = \mathbf{L}_\mathbf{s}^T{}_{|\mathbf{s}=\mathbf{s}(t)}$$

- $\widehat{\mathbf{L}_\mathbf{s}^T}^+$ is the pseudo inverse of $\widehat{\mathbf{L}_\mathbf{s}^T}$. It is a $6 \times k$ matrix such that $\widehat{\mathbf{L}_\mathbf{s}^T}^+ \widehat{\mathbf{L}_\mathbf{s}^T} = \mathbb{I}_6$.

When the selected visual features do not constrain all the n system dof, it is possible to combine a secondary task $\mathbf{e}_2$ to the visual task. When $\mathbf{e}_2$ is a simple trajectory following (such as for example $x(t) - x(0) - Vt = 0$), the new control law is given by:

$$\mathcal{T}_c = -\lambda\ \mathbf{W}^+ \mathbf{W} \widehat{\mathbf{L}_{\mathbf{s}}^T}^+ (\mathbf{s}(t) - \mathbf{s}^*) - (\mathbb{I}_n - \mathbf{W}^+ \mathbf{W})\ \frac{\partial \mathbf{e}_2}{\partial t} \tag{2}$$

where:

- $\mathbf{W}$ is a full rank matrix such that Ker $\mathbf{W}$ = Ker $\mathbf{L}_{\mathbf{s}}^T$. $\mathbf{W}$ is a $m \times n$ matrix (where m is the rank of $\mathbf{L}_{\mathbf{s}}^T$) ;

2 Positioning with respect to a square

We consider as visual features the x and y coordinates in the image of the four points composing the square. We recall that the interaction matrix related to the coordinates of an image point is given by: On rappelle que dans le cas d'un point de coordonnées (X, Y) dans le plan image et de profondeur z dans le repère de la caméra, la matrice d'interaction est donnée par :

$$\mathbf{L}_{xy}^T(x, y, Z) = \begin{pmatrix} -1/Z & 0 & x/Z & xy & -(1+x^2) & y \\ 0 & -1/Z & y/Z & 1+y^2 & -xy & -x \end{pmatrix}$$

where Z is the range of the corresponding 3D point. The desired pose between the camera and the square is such that the square appears in the image as a centered square.

1. Complete the interaction matrix in file *sp_int.c*.
2. Program the control law (1) in procedure *calc_comm_vision* in file *sp_comm.c*.
3. Explain why the control law given by (2) is equivalent to (1) in the considered case: what is the most simple form for $\mathbf{W}$ here ? What is the value of $\mathbb{I}_n - \mathbf{W}^+\mathbf{W}$? Is it possible to consider a secondary task?
4. Using software *av*, look at the obtained plots ($\mathbf{s}(t) - \mathbf{s}^*$, $\|\mathbf{s}(t) - \mathbf{s}^*\|$, $\mathcal{T}_c$ and the trajectory in the image) for different values of gain λ and for the two possible choices for $\widehat{\mathbf{L}_{\mathbf{s}}^T}$. What is necessary to use $\mathbf{L}_{\mathbf{s}}^T{}_{|\mathbf{s}=\mathbf{s}(t)}$?

3 Tracking the square

The square is now moving in successive different directions and with unknowns amplitude.

1. Using only gain λ, is it possible to suppress tracking errors ?
2. Program the integral term in file *sp_comm.c* to use the control law:

$$\mathcal{T}_{c(k)} = -\lambda \, \mathbf{e}_{(k)} - \mu \sum_{i=0}^{k} \mathbf{e}_{(i)}$$

3. Tune as best as possible gains λ and μ.

4 Positioning wrt 3 straight lines

The considered scene is now composed of 3 parallel straight lines. If we use parameters θ and ρ to describe a 2D straight line (such that $X\cos\theta + Y\sin\theta - \rho = 0$ for all points belonging to the straight line), we recall that:

$$\begin{array}{ll} L_\theta^T = [& \lambda_\theta \cos\theta \quad \lambda_\theta \sin\theta \quad -\lambda_\theta \rho \quad -\rho\cos\theta \quad -\rho\sin\theta \quad -1 \;] \\ L_\rho^T = [& \lambda_\rho \cos\theta \quad \lambda_\rho \sin\theta \quad -\lambda_\rho \rho \quad (1+\rho^2)\sin\theta \quad -(1+\rho^2)\cos\theta \quad 0 \;] \end{array}$$

where λ_θ and λ_ρ depend of the position of the corresponding 3D straight line.

1. Program the interaction matrix in file *sp_int.c*.
2. Is it now possible to consider a secondary task ? Complete file *sp_comm.c* to implement (2).
3. Comment the effect of $\mathbb{I}_n - \mathbf{W}^+\mathbf{W}$ on the components of $\frac{\partial \mathbf{e}_2}{\partial t}$. Choose the most simple form for $\frac{\partial \mathbf{e}_2}{\partial t}$ in order that the camera moves forward at 1 m/s. Are motions in other directions possible ?

5 Positioning wrt a cylinder

Program the interaction matrix in file *sp_int.c* and choose the components of $\frac{\partial \mathbf{e}_2}{\partial t}$ such that the camera has a constant velocity $V_x = 0.5$ m/s and $V_y = 0.5$ m/s. What is now the effect of $\mathbb{I}_n - \mathbf{W}^+\mathbf{W}$?