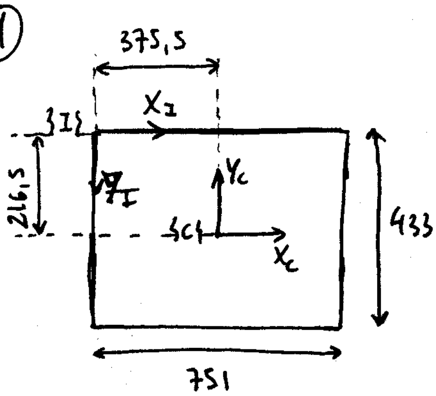
SOLUCIÓ EXAMEN DE ROBÒTICA GENER 2004

1



cada píxel fa $12\times12\,\mu m = 12\cdot10^{-3}\times12\cdot10^{-3}$ mm

$$\left.\begin{aligned} X_c &= (X_I-375{,}5)\cdot12\cdot10^{-3}\ \text{mm}\\ Y_c &= -(Y_I-216{,}5)\cdot12\cdot10^{-3}\ \text{mm}\\ Z_c &= 0\end{aligned}\right\}$$

Per un píxel de coordenades $^{I}P=(X_I,Y_I)$

en forma matricial,

$$\begin{pmatrix}X_c\\Y_c\\Z_c\\1\end{pmatrix}=\underbrace{\begin{pmatrix}12\cdot10^{-3}&0&0&-375{,}5\cdot12\cdot10^{-3}\\0&-12\cdot10^{-3}&0&+216{,}5\cdot12\cdot10^{-3}\\0&0&1&0\\0&0&0&1\end{pmatrix}}_{^{C}T_I}\begin{pmatrix}X_I\\Y_I\\0\\1\end{pmatrix}$$

Segons la figura, es pot veure que $^{L}T_c=\begin{pmatrix}0&0&1&16\\1&0&0&400\\0&1&0&0\\0&0&0&1\end{pmatrix}$

i per tant, $^{L}T_I={}^{L}T_c\cdot{}^{C}T_I=\begin{pmatrix}0&0&1&16\\12\cdot10^{-3}&0&0&400-375{,}5\cdot12\cdot10^{-3}\\0&-12\cdot10^{-3}&0&216{,}5\cdot12\cdot10^{-3}\\0&0&0&1\end{pmatrix}$

es pot trobar, doncs $^{L}P={}^{L}T_I\cdot{}^{I}P={}^{L}T_I\cdot\begin{pmatrix}120\\310\\0\\1\end{pmatrix}=\begin{pmatrix}16\\396{,}934\\-1{,}122\\1\end{pmatrix}$

també es pot trobar $^{L}F={}^{L}T_c\cdot{}^{C}F={}^{L}T_c\cdot\begin{pmatrix}0\\0\\-16\\1\end{pmatrix}=\begin{pmatrix}0\\400\\0\\1\end{pmatrix}$

Per tant, $^{L}\begin{pmatrix}V_x\\V_y\\V_z\end{pmatrix}=\begin{pmatrix}16\\396{,}934\\-1{,}122\end{pmatrix}-\begin{pmatrix}0\\400\\0\\1\end{pmatrix}=\begin{pmatrix}16\\-3{,}066\\-1{,}22\end{pmatrix}\Longrightarrow\boxed{^{L}\begin{pmatrix}r_x\\r_y\\r_z\end{pmatrix}=\lambda\cdot\begin{pmatrix}16\\-3{,}066\\-1{,}22\end{pmatrix}+\begin{pmatrix}0\\400\\0\end{pmatrix}}$

EQUACIÓ DE LA RECTA QUE PASSA PER ^{C}F i ^{I}P, referida al S.C. $\{L\}$

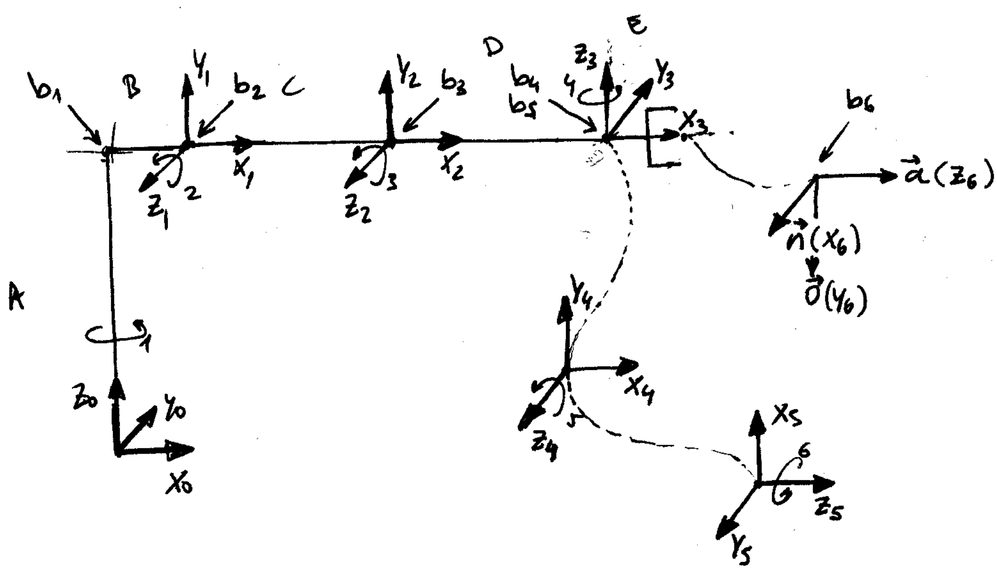
eq. del pla: $y=0$

$$\left.\begin{aligned} r_x &= \lambda \cdot 16 \\ r_y &= -\lambda \cdot 3{,}066 + 400 \\ r_z &= -\lambda \cdot 1{,}22 \\ y &= 0 \end{aligned}\right\} \Longrightarrow \quad 0 = -\lambda \cdot 3{,}066 + 400$$

$$\lambda = \frac{400}{3{,}066} = 130{,}463$$

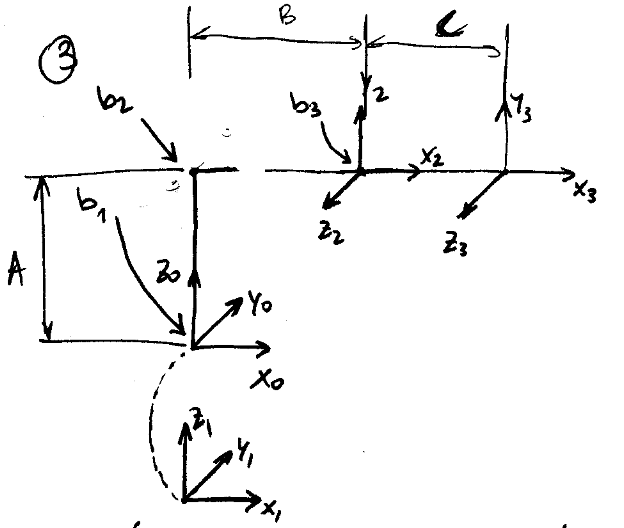
$$\boxed{\begin{aligned} r_x &= 2087{,}408 \\ r_y &= 0 \\ r_z &= -159{,}165 \end{aligned}} \Longrightarrow \quad {}^{L}P = \begin{pmatrix} 2087{,}408 \\ 0 \\ -159{,}165 \\ 1 \end{pmatrix}$$

2



Art.	θ	d	a	α	HOME
1	q_1	A	B	90°	0°
2	q_2	0	C	0°	0°
3	q_3	0	D	-90°	0°
4	q_4	0	0	90°	0°
5	q_5	0	0	90°	90°
6	q_6	E	0	0°	90°

3



Art	θ	d	a	α	HOME
1	q_1	0	0	0°	0°
2	0°	q_2	B	90°	A
3	q_3	0	C	0°	0°

$$ {}^0A_1 = \begin{pmatrix} C_1 & -S_1 & 0 & 0 \\ S_1 & C_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^1A_2 = \begin{pmatrix} 1 & 0 & 0 & BC_2 \\ 0 & 0 & -1 & BS_2 \\ 0 & 1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad {}^2A_3 = \begin{pmatrix} C_3 & -S_3 & 0 & C\cdot C_3 \\ S_3 & C_3 & 0 & C\cdot S_3 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ {}^0A_2 = \begin{pmatrix} C_1 & 0 & S_1 & B(C_1C_2 - S_1S_2) \\ S_1 & 0 & -C_1 & B(S_1C_2 + C_1S_2) \\ 0 & 1 & 0 & q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \cdot {}^2A_3 = \begin{pmatrix} C_1C_3 & -C_1S_3 & S_1 & C\cdot C_1\cdot C_3 + B(C_1C_2 - S_1S_2) \\ S_1C_3 & -S_1S_3 & -C_1 & C\cdot S_1\cdot C_3 + B(S_1C_2 + C_1S_2) \\ S_3 & C_3 & 0 & C\cdot S_3 + q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

$$ \| $$

$$ {}^0A_3 = {}^RT_H $$

$$ {}^RT_H = \begin{pmatrix} C_1C_3 & -C_1S_3 & S_1 & C\cdot C_1C_3 + B\cdot C_{12} \\ S_1C_3 & -S_1S_3 & -C_1 & C\cdot S_1C_3 + B\,S_{12} \\ S_3 & C_3 & 0 & C\cdot S_3 + q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} \xrightarrow{\theta_2 = 0°} $$

$$ \begin{pmatrix} C_1C_3 & -C_1S_3 & S_1 & C\cdot C_1C_3 + BC_1 \\ S_1C_3 & -S_1S_3 & -C_1 & C\,S_1C_3 + BS_1 \\ S_3 & C_3 & 0 & C\cdot S_3 + q_2 \\ 0 & 0 & 0 & 1 \end{pmatrix} $$

(4) Vector de configuració

$$w = \begin{pmatrix} a_1 C_1 + a_2 C_{1-2} \\ a_1 S_1 + a_2 S_{1-2} \\ d_1 - q_3 - d_4 \\ 0 \\ 0 \\ -e^{q_4/\pi} \end{pmatrix}$$

del dibuix, es pot extreure $\boxed{q_4 = 0^\circ}$

Cal dir que aquest robot no té articulació de ROLL, i per tant els vectors x_3 i n sempre seràn paral·lels ($q_4 = 0^\circ$).

Per tant, $w = \begin{pmatrix} a_1 C_1 + a_2 C_{1-2} \\ a_1 S_1 + a_2 S_{1-2} \\ d_1 - q_3 - d_4 \\ 0 \\ 0 \\ -1 \end{pmatrix}$ $\boxed{q_3 = d_1 - w_3 - d_4}$

$$w_1^2 + w_2^2 = (a_1 c_1 + a_2 c_{1-2})^2 + (a_1 s_1 + a_2 s_{1-2})^2 = \underbrace{a_1^2 c_1^2 + a_2^2 c_{1-2}^2 + 2a_1 a_2 c_1 c_{1-2}}_{w_1^2} +$$

$$+ \underbrace{a_1^2 s_1^2 + a_2^2 s_{1-2}^2 + 2a_1 a_2 s_1 s_{1-2}}_{w_2^2} = a_1^2 (\underbrace{c_1^2 + s_1^2}_{1}) + a_2^2 (\underbrace{c_{1-2}^2 + s_{1-2}^2}_{1}) + 2a_1 a_2 (c_1 c_{1-2} + s_1 s_{1-2}) =$$

$$= a_1^2 + a_2^2 + 2a_1 a_2 (c_1 c_{1-2} + s_1 s_{1-2}) = a_1^2 + a_2^2 + 2a_1 a_2 (c_1 [c_1 c_2 + s_1 s_2] + s_1 [s_1 c_2 - c_1 s_2]) =$$

$$= a_1^2 + a_2^2 + 2a_1 a_2 (c_1^2 c_2 + \cancel{c_1 s_1 s_2} + s_1^2 c_2 - \cancel{s_1 c_1 s_2}) = a_1^2 + a_2^2 + 2a_1 a_2 c_2 (\underbrace{s_1^2 + c_1^2}_{1}) = \underline{\underline{a_1^2 + a_2^2 + 2a_1 a_2 c_2}}$$

per tant, $$c_2 = \frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2a_1 a_2} \Longrightarrow \boxed{\theta_2 = \arccos\left(\frac{w_1^2 + w_2^2 - a_1^2 - a_2^2}{2a_1 a_2}\right)}$$

* Cal notar que és impossible obtenir una expressió per s_1, i per tant no es pot resoldre per <u>atan2</u>

* Cal notar també la semblança del resultat amb la resolució pel mètode geomètric.

Desenvolupant w_1 i w_2, tenim

$$a_1c_1 + a_2[c_1c_2 + s_1s_2] = w_1$$
$$a_1s_1 + a_2[s_1c_2 - c_1s_2] = w_2$$

$$a_1c_1 + a_2c_1c_2 + a_2s_1s_2 = w_1 \;; \quad c_1(a_1a_2c_2) + a_2s_1s_2 = w_1 \;; \quad c_1 = \frac{w_1 - a_2s_1s_2}{a_1a_2c_2}$$

substituint a w_2

$$a_1s_1 + a_2s_1c_2 - \cancel{a_2}\frac{w_1 - a_2s_1s_2}{a_1\cancel{a_2}c_2}\cdot s_2 = w_2$$

$$s_1\left(a_1 + a_2c_2 + \frac{a_2}{a_1}tg_2\cdot s_2\right) - \frac{w_1}{a_1}tg_2 = w_2$$

$$\boxed{s_1 = \frac{w_2 + \frac{w_1}{a_1}tg_2}{a_1 + a_2c_2 + \frac{a_2}{a_1}tg_2\cdot s_2}}$$

substituint a l'expressió trobada per c_1, es troba el valor de q_1 fent:

$$q_1 = atan2(s_1, c_1)$$

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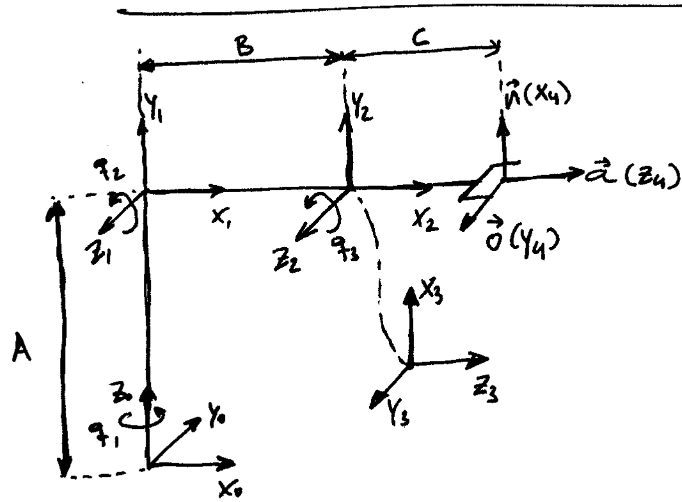
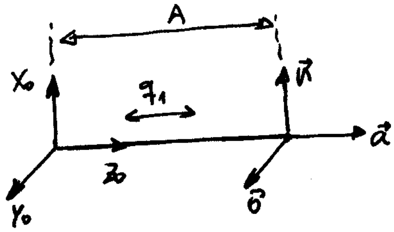
⑤

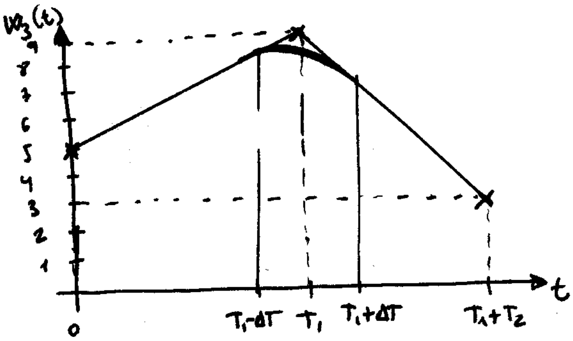


Art.	θ	d	a	α	HOME
1	q_1	A	0	90°	0°
2	q_2	0	B	0°	0°
3	q_3	0	0	90°	90°
4	q_4	C	0	0°	0°

(6)



Art.	θ	d	a	α	HOME
1	0	q_1	0	0	A



$T_1 = 3s$
$T_2 = 4s$
$\Delta T = 1s$

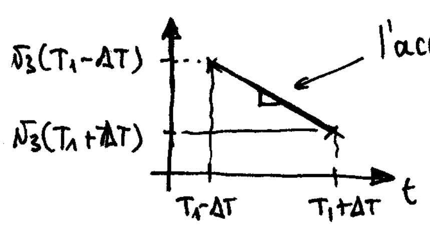
a)

$$v_3(T_1-\Delta T)_- = \dot{w}_3(T_1-\Delta T)_- = \frac{\Delta w_3^1}{T_1}$$

$$v_3(T_1+\Delta T)_+ = \dot{w}_3(T_1+\Delta T)_+ = \frac{\Delta w_3^2}{T_2}$$

de l'expressió de $w_3(t)$

En la interpolació lineal amb xamfrà parabòlic, la posició en l'interval del xamfrà $t \in [T_1-\Delta T, T_1+\Delta T]$ té una expressió de 2on ordre, per tant l'acceleració és constant i la velocitat és lineal:



l'acceleració és el pendent

$$a = \frac{v_3(T_1+\Delta T) - v_3(T_1-\Delta T)}{\cancel{T_1}+\Delta T - \cancel{T_1} + \Delta T} = \frac{\frac{\Delta w_3^2}{T_2} - \frac{\Delta w_3^1}{T_1}}{2\Delta T} =$$

$$= \frac{\frac{T_1 \Delta w_3^2 - T_2 \Delta w_3^1}{T_1 T_2}}{2\Delta T} = \boxed{\frac{T_1 \Delta w_3^2 - T_2 \Delta w_3^1}{2 T_1 T_2 \Delta T}}$$

Substituint valors,

$T_1 = 3$
$T_2 = 4$
$\Delta T = 1$
$\Delta w_3^2 = w_3^2 - w_3^1 = 3 - 9 = -6$
$\Delta w_3^1 = w_3^1 - w_3^0 = 9 - 5 = 4$

$$\boxed{a = \frac{3 \cdot (-6) - 4 \cdot 4}{2 \cdot 3 \cdot 4 \cdot 1} = -1{,}4167 \ cm/s^2}$$

b)

a l'interval $t \in [0,2]$,

$$\boxed{v_3 = \dot{w}_3(t) = \frac{\Delta w_3^1}{T_1} = \frac{w_3^1 - w_3^0}{T_1} = \frac{9-5}{3} = 1{,}\hat{3}\ \text{cm/s}}$$

a l'interval $t \in [4,7]$,

$$\boxed{v_3 = \dot{w}_3(t) = \frac{\Delta w_3^2}{T_2} = \frac{w_3^2 - w_3^1}{T_2} = \frac{3-9}{4} = -1{,}5\ \text{cm/s}}$$

c)

$$\boxed{w_3(3) = \frac{-1{,}4167}{2}(3-3+1)^2 + \frac{4(3-3)}{3} + 9 = 8{,}2917}$$